Thermally Interacting Multiple Boreholes with Variable Heating Strength

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Abstract

A line source of finite length in a three dimensional domain is used to model the conduction of heat in the soil surrounding multiple boreholes. In order to determine the heat flux boundary condition, the analytical quasi-three-dimensional solution to the heat transfer problem of the U-tube configuration inside the borehole is used. This solution takes into account the variation in heating strength along the borehole length due to the temperature variation of the fluid flowing in the U-tube. Thus, critical depths at which thermal interaction occurs can be determined.

1 Introduction

The use of geothermal energy is advantageous in many cases due to its efficiency. However, little research is available to guide regulatory agencies and industry towards designs and installations that maximize their sustainability. One potential hindrance to the sustainability of these systems at their design efficiency is the thermal loss from the system itself, which can affect adjacent systems and the surrounding ground. Studies show that interference effects are present in some installed geothermal systems. The influence of these systems on each other implies that they have a spacing that is smaller than the threshold spacing for such systems to avoid thermal interactions, and indicates that there is a limit to the density of geothermal development that can occur in a given region of the ground.

Many studies in the area of geothermal energy have focused on modeling single ground boreholes, most of which are based on analytical methods (Jun et al. 2009, Eskilson 1987, Hellström 1991, Zobel & Ingersoll 1954, Hart & Couvillion 1986, Ingersoll & Plass 1948, Lamarche & Beauchamp 2007, Kavanaugh 1992, Hikari et al. 2004, Zeng et al. 2002, Diao et al. 2004, Yang et al. 2009, Bernier et al. 2004, Kavanaugh 1995, Bandyopadhya et al. 2008) and/or numerical methods (Mei & Baxter 1986, Yavuzturk 1999, Yavuzturk et al. 1999, Yavuzturk & Spitler 2001, Muraya 1995, Kavanaugh 1985, Rottmayer et al. 1997, Lee & Lam 2008, Li & Zheng 2009, He et al. 2009, Fang et al. 2002). The models vary in the way heat conduction in the soil is solved and heat transfer outside of the boreholes is coupled to the heat transfer inside of the borehole, and in the way the methods are accelerated.

The key limitation in most of the previous studies is the assumption of constant and uniform strength of the heat input from the borehole to the ground, either when the borehole is assumed as a cylinder or when it is further simplified to a line source of heat. Duan et al. (2007) and Duan and Naterer (2007) study the transient heat conduction from a buried power transmission line tower. In their study they formulated the problem of ground heat transfer with a line source of heat with varying heating strength along its length. However, the physical nature of their problem involved pure conduction along a buried rod in the ground which makes it somewhat different from borehole analysis. Furthermore, the potential existence of thermal interaction among multiple boreholes is identified in the literature, but not formulated, and the affecting parameters have not been assessed in detail. This is another key limitation in the past studies in the area of ground heat exchangers. In order to model interacting borehole systems, Koohi-Fayegh & Rosen (2012) evaluate the temperature response in the soil surrounding multiple boreholes numerically. They assume that the heat flux from the borehole wall is constant and, therefore, that heat conduction in the direction of the borehole length is negligible for a major part of the solution domain. Later, Koohi-Fayegh & Rosen (2011) perform a numerical finite volume analysis to study the thermal interaction among boreholes with varying heat input into the ground. They show that due to the higher heating strength at the top end of the boreholes, thermal interaction at this depth is at its highest value.

The current study addresses the shortcomings of the past studies by considering a variable heat flux along walls of multiple boreholes by coupling the problem to the heat transfer problem inside the boreholes. An analytical solution to the line source with finite length is used to model the conduction of heat in the soil surrounding boreholes. In order to determine the heat flux boundary condition, the analytical quasi-three-dimensional solution to the heat transfer problem of the U-tube configuration inside the borehole (Zeng et al. 2003a) is used. This solution takes into account the variation in heating strength along the borehole length due to the temperature variation of the fluid running in the U-tube. Thus, critical depths at which thermal interaction occurs can be determined.

2 Methods

To examine the existence of thermal interaction among multiple boreholes and their possible negative effects on the design performance of the existing nearby boreholes, the transient conduction of heat in the soil surrounding these systems needs to be studied in order to evaluate the temperature rise and the heat flows in the soil surrounding the boreholes. Representation of heat flows to and from the system based in this simulation can serve as inputs into large scale ground water models.

Variable heat flux model

A quasi-three-dimensional model was proposed by Zeng et al. (2003a, 2003b) taking into account the fluid axial convective heat transfer and thermal “short-circuiting” among U-tube legs. Being minor in order, the conductive heat flow in the grout and ground in the axial direction, however, is still neglected to keep the model concise and analytically manageable. The energy balance equations for up-flow and down-flow of the circulating fluid can be written as

$$\begin{aligned} -\dot{m}c_p \frac{dT_{f1}}{dz} &= \frac{T_{f1}-T_b}{R_1^\Delta} + \frac{T_{f1}-T_{f2}}{R_{12}^\Delta} \\ \dot{m}c_p \frac{dT_{f2}}{dz} &= \frac{T_{f2}-T_b}{R_2^\Delta} + \frac{T_{f2}-T_{f1}}{R_{12}^\Delta} \end{aligned} \qquad (0 \le z \le H) \tag{1}$$

where T_{f1} , T_{f2} and T_b are the temperatures of the fluid running downwards, the fluid running upwards and borehole wall, respectively, and

$$R_1^\Delta = \frac{R_{11}R_{22}-R_{12}^2}{R_{22}-R_{12}}, \quad R_2^\Delta = \frac{R_{11}R_{22}-R_{12}^2}{R_{11}-R_{12}}, \quad \text{and} \quad R_{12}^\Delta = \frac{R_{11}R_{22}-R_{12}^2}{R_{12}} \tag{2}$$

Here, R_{11} and R_{22} are the thermal resistance between the circulating fluid and the borehole wall, and R_{12} is the resistance between the pipes (Figure 1). In most engineering applications, the configuration of the U-tube in the borehole may be assumed symmetric, and here it is assumed that $R_{22}=R_{11}$. Therefore,

$$R_1^{\Delta} = R_2^{\Delta} = R_{11} + R_{12}, \quad R_{12}^{\Delta} = \frac{R_{11}^2 - R_{12}^2}{R_{12}} \tag{3}$$

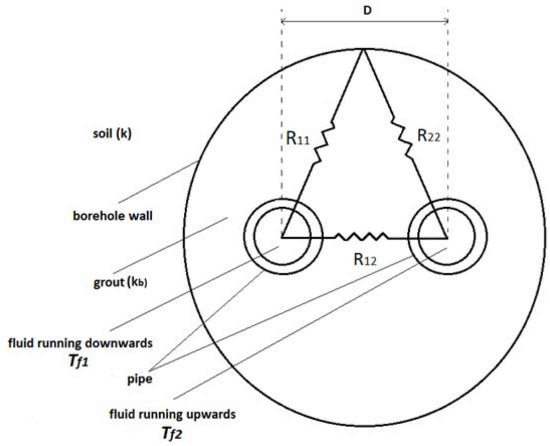


Figure 1: Thermal resistances in the borehole.

The steady-state conduction problem in the borehole cross-section was analyzed in detail by Hellström (1991) with the line source and multiple approximations. The line-source assumption results in the following solution:

$$\begin{aligned} R_{11} &= \frac{1}{2\pi k_b}\left[\ln\left(\frac{r_b}{r_p}\right) + \frac{k_b - k}{k_b + k}.\ln\left(\frac{r_b^2}{r_b^2 - D^2}\right)\right] + R_p \\ R_{12} &= \frac{1}{2\pi k_b}\left[\ln\left(\frac{r_b}{2D}\right) + \frac{k_b - k}{k_b + k}.\ln\left(\frac{r_b^2}{r_b^2 + D^2}\right)\right] \end{aligned} \tag{4}$$

where r_b, r_p, k_b, k, D and R_p are the radius of the boreholes, radius of the pipe, the grout thermal conductivity, the soil thermal conductivity, the distance between the pipes in the borehole, and the thermal resistance of conduction in the pipe, respectively (Figure 1).

The following boundary conditions are applied to the governing equations:

$$\begin{aligned} z = 0, &\quad T_{f1} = T_f' \\ z = H, &\quad T_{f1} = T_{f2} \end{aligned} \tag{5}$$

where T_f' is the temperature of the fluid entering the U-tube and

Zeng et al. (2003a) formulate the temperature profiles of the fluids flowing in the U-pipes in the boreholes:

$$\begin{aligned} \Theta_1(Z) &= \cosh(\beta Z) - \frac{1}{\sqrt{1-P^2}}\left[1 - P\frac{\cosh(\beta) - \sqrt{\frac{1-P}{1+P}}\sinh(\beta Z)}{\cosh(\beta) + \sqrt{\frac{1-P}{1+P}}\sinh(\beta)}\right].\sinh(\beta Z) \\ \Theta_2(Z) &= \frac{\cosh(\beta) - \sqrt{\frac{1-P}{1+P}}\sinh(\beta)}{\cosh(\beta) + \sqrt{\frac{1-P}{1+P}}\sinh(\beta)}\cosh(\beta Z) + \frac{1}{\sqrt{1-P^2}}\left[\frac{\cosh(\beta) - \sqrt{\frac{1-P}{1+P}}\sinh(\beta)}{\cosh(\beta) + \sqrt{\frac{1-P}{1+P}}\sinh(\beta)} - P\right].\sinh(\beta Z \end{aligned} \tag{6}$$

where the dimensionless parameters are defined as

$$\Theta = \frac{T_f(z) - T_b}{T_f' - T_b}, \quad Z = \frac{z}{H}, \quad P = \frac{R_{12}}{R_{11}} \tag{7}$$

$$\beta = \frac{H}{\dot{m}c_p\sqrt{(R_{11} + R_{12})(R_{11} - R_{12})}}$$

The heat transferred to the soil from each of the pipes in the borehole can be obtained from Eq. (1). In this equation we only take the first term on the right hand side.

$$q''(z) = \frac{T_{f1}(z) - T_b}{R_1^\Delta} + \frac{T_{f2}(z) - T_b}{R_2^\Delta} \tag{8}$$

where $R_1^\Delta = R_2^\Delta$ (Eq. (4)). Using the dimensionless parameters introduced in Eq. (6), Eq. (8) can be rewritten in terms of the dimensionless parameters:

$$q''(Z) = (T_f' - T_b)\left[\frac{\Theta_1(Z)}{R_1^\Delta} + \frac{\Theta_2(Z)}{R_2^\Delta}\right] \tag{9}$$

Assuming that the heat is dissipated symmetrically in the soil around each borehole, Eq. (9) can be written in the following form:

$$q'(Z) = \pi D_b (T_f' - T_b)\left[\frac{\Theta_1(Z)}{R_1^\Delta} + \frac{\Theta_2(Z)}{R_2^\Delta}\right] \tag{10}$$

This is the spatial distribution of the heating strength along the rod. In contrast to past studies, this heating strength varies along the rod and is not constant. Note that the variable heat source (VHS) model has made certain simplifying assumptions, such as constant ground temperature.

In order to compare the results gained by constant heat flux model with the results gained by the VHS model, an equivalent inlet temperature ($T_f' = 290.6\,\mathrm{K}$) for the VHS model, resulting in the same total heat conduction in the soil, is assumed.

Analytical approach

A three-dimensional model of transient conduction of heat in the soil around multiple ground heat exchangers is presented in this section. A domain consisting of two vertical borehole heat exchangers having a distance of $2h$ from each other is considered (Figure 2).

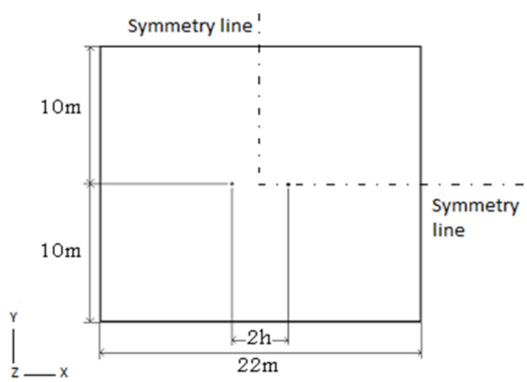


Figure 2: Two-dimensional view of the solution domain: horizontal cross sections (*xy*) at the borehole mid-length (*Z* = 0.5).

It is assumed that the dominant mode of heat transfer in the soil is conduction. The general heat conduction equation in cylindrical coordinates appears in the following form:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}_{gen}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{11}$$

where t is the time from the start of operation, α is the thermal diffusivity of soil, and T is the temperature of the ground. The first two terms on the left side of Eq. (11) are the heat flux components in the radial (r) direction, the third and the fourth terms are related to the circumferential (φ) and axial (z) directions, respectively, and the fifth term relates to the heat generated in the control volume. The right side of Eq. (1) represents the transient effects of heat conduction.

The model of Zeng et al. (2002) establishes the transient response at any point in the ground, subject to a constant line heat source in the rod. However, the previous analysis has shown that the heating strength varies with depth. Thus, Zeng's model can be extended to this case by integrating the heating strength over the depth of the rod. The temperature response at any point in the semi-infinite medium will be calculated for a point, $P(q,z)$, in the medium. Duan et al. (2007) and Duan and Naterer (2007) extended Zeng's model (2002) for the case of a buried rod. They formulated the temperature profile in the soil around the rod.

$$\theta\left(\overline{R},Z,Fo\right)=\int_0^1\frac{q'\left(\overline{H}\right)}{4k\pi}\left[\frac{erfc\left(\dfrac{\sqrt{\overline{R}^2+\left(Z-\overline{H}\right)^2}}{2\sqrt{Fo}}\right)}{\sqrt{\overline{R}^2+\left(Z-\overline{H}\right)^2}}-\frac{erfc\left(\dfrac{\sqrt{\overline{R}^2+\left(Z+\overline{H}\right)^2}}{2\sqrt{Fo}}\right)}{\sqrt{\overline{R}^2+\left(Z+\overline{H}\right)^2}}\right]d\overline{H} \tag{12}$$

where $\theta = T - T_0$, $\overline{H}=\frac{h_z}{H}$, $\overline{R}=\frac{r}{H}$, $Fo=\frac{\alpha t}{H^2}$ and $q'(H')$ is the heating strength per unit length.

Using this procedure for the case of a vertical boreholes containing U-pipes with running fluid, the heating strength formulated for the case of variable heating strength (Eq. (10)) is substituted in Eq. (12) to obtain the temperature rise in the soil surrounding a borehole.

For the case of multiple boreholes, since the conduction equation is linear, the temperature response in the soil can be calculated by supersposing the temperature rise in the soil caused by each single borehole. Koohi-Fayegh & Rosen (2012) examined the validity of superposition method in thermal response in the soil surrounding multiple boreholes by comparing the superposition results of the line source theory with results obtained by a finite volume numerical method. It was shown that the results of the two methods agree well and the effect of the temperature rise due to one borehole on the thermal performance of other boreholes can be neglected. Therefore, the temperature response in the soil surrounding a borehole system of n boreholes can be calculated by superposing the temperature response evaluated by each borehole in Eq. (12):

$$\theta\left(R,Z,Fo\right)=\sum_1^n\int_0^1\frac{q_i'\left(\overline{H}\right)}{4k\pi}\left[\frac{erfc\left(\dfrac{\sqrt{\overline{R}_i^2+\left(Z_i-\overline{H}\right)^2}}{2\sqrt{Fo}}\right)}{\sqrt{\overline{R}_i^2+\left(Z_i-\overline{H}\right)^2}}-\frac{erfc\left(\dfrac{\sqrt{\overline{R}_i^2+\left(Z_i+\overline{H}\right)^2}}{2\sqrt{Fo}}\right)}{\sqrt{\overline{R}_i^2+\left(Z_i+\overline{H}\right)^2}}\right]dH' \tag{13}$$

where q_i' is the heat flow rate per unit length of Borehole i (Figure 3) and

$$\overline{R}_i = \frac{\sqrt{(x - l_i)^2 + (y - w_i)^2}}{H} \tag{14}$$

where l_i and w_i are distances of boreholes i along x and y directions, respectively. For the case of multiple boreholes shown in Figure (2), Eq. (13) can be simplified to

$$\begin{aligned}\theta(\overline{R}, Z, Fo) = &\int_0^1 \frac{q'(\overline{H})}{4k\pi}\left[\frac{erfc\left(\frac{\sqrt{\overline{R}_1^2 + (Z - \overline{H})^2}}{2\sqrt{Fo}}\right)}{\sqrt{\overline{R}_1^2 + (Z - \overline{H})^2}} - \frac{erfc\left(\frac{\sqrt{\overline{R}_1^2 + (Z + \overline{H})^2}}{2\sqrt{Fo}}\right)}{\sqrt{\overline{R}_1^2 + (Z + \overline{H})^2}}\right]d\overline{H} \\ &+ \int_0^1 \frac{q'(\overline{H})}{4k\pi}\left[\frac{erfc\left(\frac{\sqrt{\overline{R}_2^2 + (Z - \overline{H})^2}}{2\sqrt{Fo}}\right)}{\sqrt{\overline{R}_2^2 + (Z - \overline{H})^2}} - \frac{erfc\left(\frac{\sqrt{\overline{R}_2^2 + (Z + \overline{H})^2}}{2\sqrt{Fo}}\right)}{\sqrt{\overline{R}_2^2 + (Z + \overline{H})^2}}\right]d\overline{H}\end{aligned} \tag{15}$$

where, as seen in Figure 3,

$$\overline{R}_1 = \frac{R_1}{H} = \frac{\sqrt{(x + h)^2 + y^2}}{H} \quad and \quad \overline{R}_2 = \frac{R_2}{H} = \frac{\sqrt{(x - h)^2 + y^2}}{H} \tag{16}$$

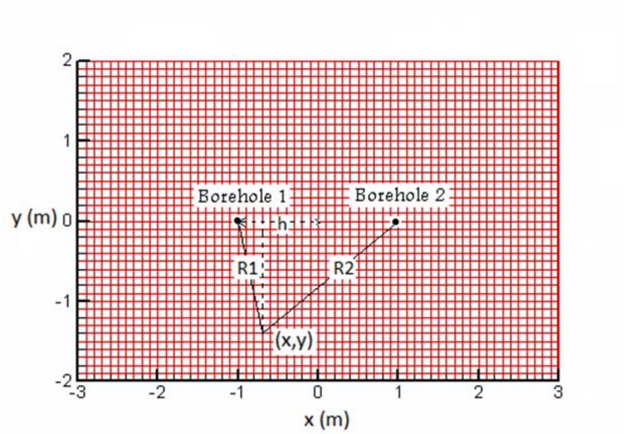


Figure 3: System geometric parameters for two boreholes at distances R_1 and R_2 from a fixed point in the surrounding soil.

Initial and boundary conditions

A uniform initial temperature of 288 K (equal to the undisturbed ground temperature) is assumed to be effective over the entire borefield. The ground surface is assumed to be isothermal and equal to the ground initial temperature. The temperature and heat flux distributions on the borehole wall cannot be decided due to the dynamic nature of the heat exchange process between the pipes in the borehole and the borehole wall. However, to simplify the current model, a constant heat flux of 10 W/m^2 on the borehole wall can be assumed since in order to study the thermal interaction between multiple boreholes, their inner dynamic heat exchange process can be of second priority compared to the heat dissipation in the soil surrounding them. As a second approach, a variable heat flux (VHS) along the borehole is calculated by defining the temperature profiles of the fluid running along the pipes in the borehole.

3 Results and discussion

In the current study, typical geometrical and thermal characteristics for the borehole and the surrounding soil are assumed (Table 1). Note that the properties of soil are approximate values for dry clay.

Table 1: Parameters of the reference borehole

(a) Inside the borehole (pipe and grout region)

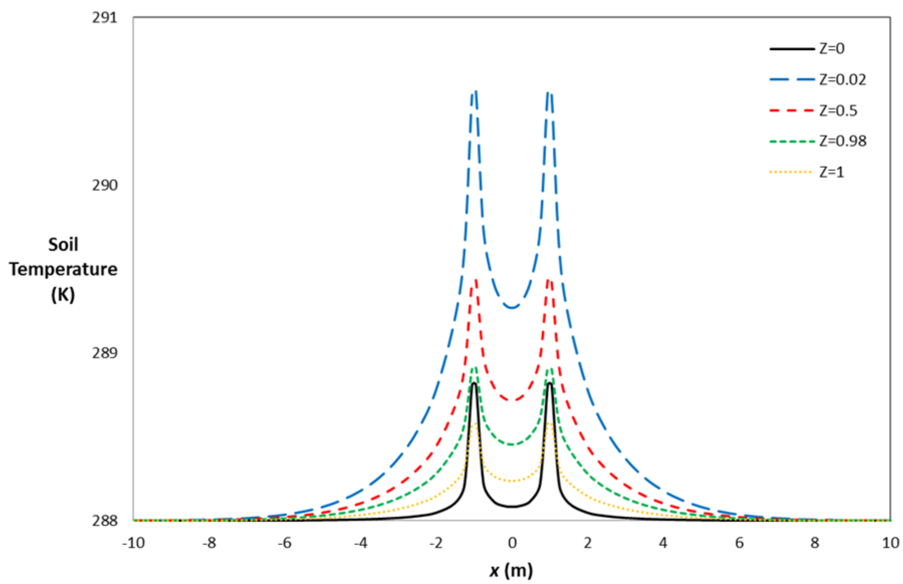
H (m)	r_b (m)	r_p (m)	D (m)	D_b (m)	β	P	k_b (W/mK)	$\dot{m}$ (kg/s)	c (J/kgK)
200	0.05	0.02	0.02	2	1.8	0.3	1	0.2	4187

(b) Outside the borehole (soil region)

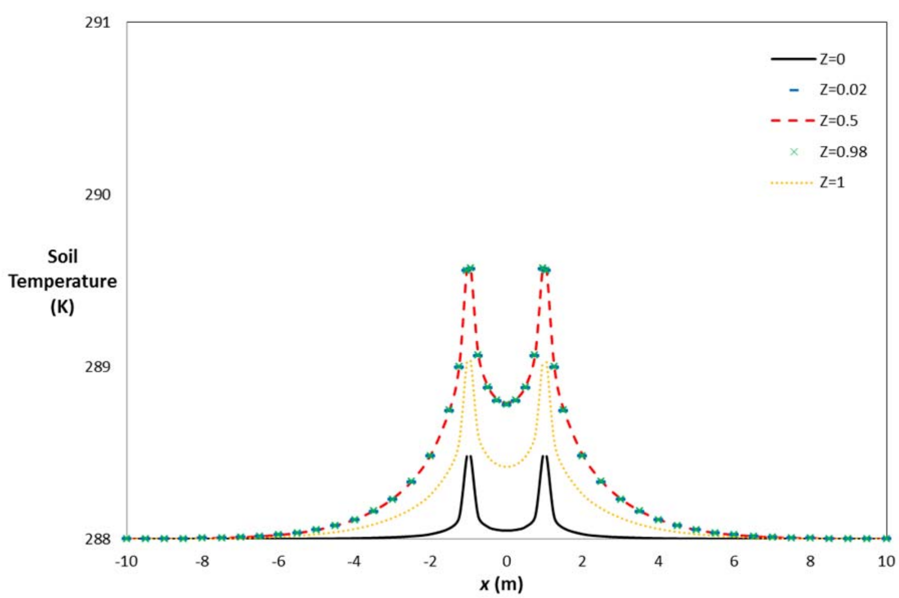
k (W/mK)	c_p (J/kgK)	ρ (kg/m^3)
1.5	1381	1200

The temperature responses of the soil around multiple boreholes evaluated by the VHS model at various borehole depths are compared in Figure 4a. It is shown that the maximum temperature rise due to thermal interaction of multiple boreholes in a six-month period of heat transfer from the borehole to the soil occurs at the top 3% of the heating length of the borehole (z = 95 m) and it decreases along the borehole length as the heat flux from the borehole wall into the soil decreases. Therefore, with the objective of limiting boreholes' operations and sizes in order to prevent their thermal interaction, the top length of the boreholes (about 3% total length) is the critical area. This is confirmed in a recent numerical study by Koohi-Fayegh & Rosen (2011) as well. The thermal interaction between the boreholes is at its minimum at the bottom of the borehole (Z = 1) where the heat flux to the soil is lowest. This is not true for the case of constant heat flux from the borehole wall to the surrounding soil along the borehole length (Figure 4b). It is seen in Figure 4b that the greatest thermal interaction occurs at top of the borehole, but remains at its maximum amount along the borehole length. For this case, the critical length of the borehole would be almost 95% of the borehole length. However, as discussed earlier, the case of constant heat flux is only a simplification to the VHS problem and does not present the problem as accurate as the VHS problem.

Another notable characteristic of Figures 4a and 4b is the decrease in the thermal interaction in the lengths of Z = 0 when one moves from Z = 0.02 towards the top end of the borehole. Specifically for the case of VHS (Figure 4a), there is higher heat flux as one moves towards the top end and one expects greater thermal interactions. In both cases, the decrease in the temperature rise in the soil around the borehole declines at the very end of borehole length, and this can be due to axial heat transfer effects which become notable only at the very ends of borehole lengths.



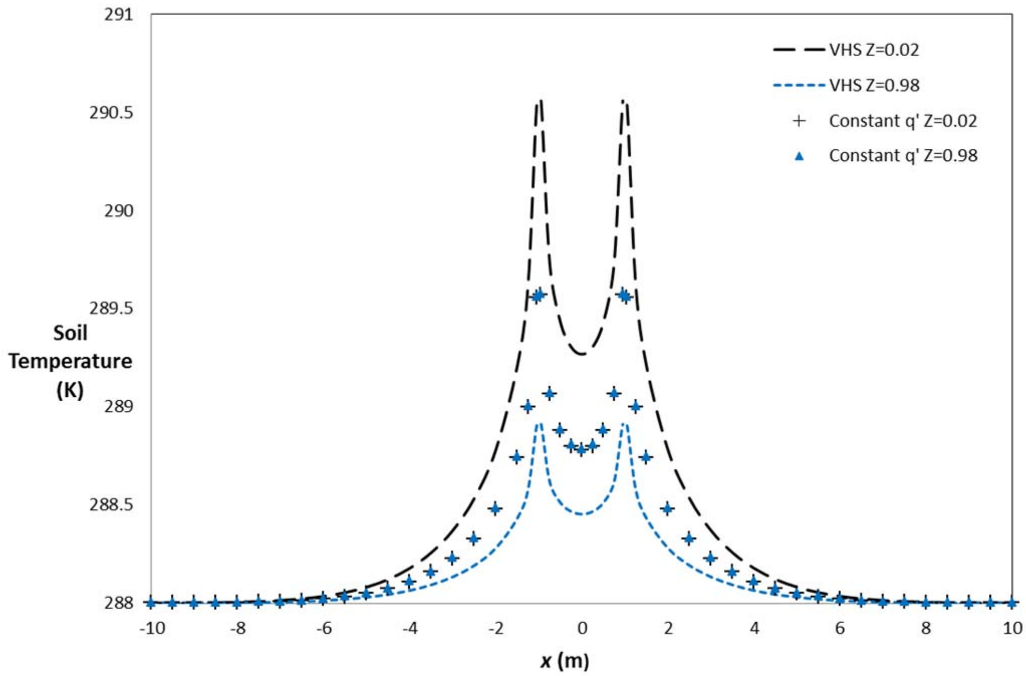
(a)



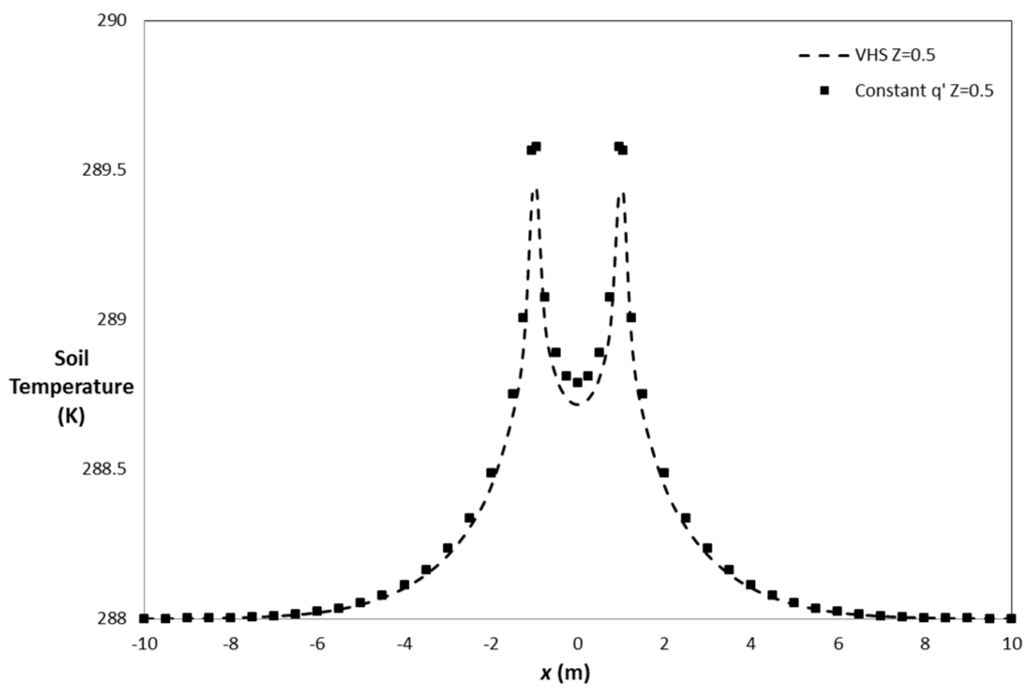
(b)

Figure 4: Soil temperature around multiple boreholes, installed at (*x*,*y*) = (1 m , 0 m) and (*x*,*y*) = (-1 m , 0 m), at *t* = 6 months, at various borehole depths for (a) VHS model, and (b) constant heat source model.

The results of the VHS model and constant heat flux model are compared in Figure 5. It is seen in Figure 5a that the assumption of line source with constant heat flow rate along borehole wall introduces numerous inaccuracies especially when dealing with the temperature rises in the soil at the very top and bottom of the borehole. Figure 5b shows that, by using the quasi-three-dimensional model to evaluate the heat transfer inside the borehole, the heat flux on the borehole is spread along the borehole in a way that the middle area remains similar to its average amount. It can be concluded that using the constant heat source method is only valid for the middle length of the boreholes and moving any further to the top or bottom of the borehole, the temperature rises evaluated become increasingly inaccurate. Quasi-three-dimensional models reveal drawbacks of two-dimensional models and are thus preferred for design and analysis of ground heat exchangers, as they provide more accurate information for performance simulation and analysis and design.



(a)



(b)

Figure 5: Comparison of soil temperature around multiple boreholes, installed at (x,y) = (1 m , 0 m) and (x,y) = (-1 m , 0 m), at t = 6 months for VHS and constant heat source models, at (a) Z = 0.02 and Z = 0.98, and (b) Z = 0.

It should be noted that the effect of temperature rise due to one borehole on the other is neglected by applying the superposition method. This effect has been examined for a two-dimensional numerical study by Koohi-Fayegh and Rosen (2012). A comparison of the results of the numerical solution with analytical results of line source theory where the superposition method is used to account for the temperature rise in the soil surrounding multiple boreholes shows that these effects are minor in comparison to the order of the temperature rise in the soil due to the individual performance of the boreholes. Since the objective in the current study is to examine at what depths the thermal interaction among boreholes creates a critical temperature rise, the focus is mostly on introducing a heat flow rate profile along the borehole length and further coupling it to the classical line source model where semi-analytical solutions can be derived. A comparison of the current results with the numerical results presented by Koohi-Fayegh & Rosen (2011) where the effects of the thermal performance of the boreholes on each other is accounted for is subject of ongoing research by the authors.

Extension of results to systems of boreholes

The methods used for calculating the temperature profiles in the soil around two boreholes can also be applied to two systems of vertical GHEs. For example, if an area of 40 m x 40 m x 200 m in the soil is occupied for one system of vertical GHEs, the ratio of system depth to its initial size is large enough to be accounted as one cylinder or line source of heat when system interactions and temperature excess around a system with larger distances are to be accounted for. The study of variable heating strength along the borehole length also accounts for the system of boreholes as well. Therefore, the parametric study on two interacting boreholes likely exhibits the same results as those for two interacting systems of boreholes. However, the assumption of constant ground temperature must be examined further in order to improve the accuracy of the proposed method.

4 Conclusions

The performance of multiple boreholes or neighbouring borehole systems and their possible thermal interactions are discussed. The effect of borehole heat flux on the transient response of multiple ground heat exchangers and their thermal interaction is described using an analytical approach. In addition, a quasi-three-dimensional model for heat transfer inside the borehole is utilised as the boundary condition for the three-dimensional transient heat transfer analysis outside the borehole in order to evaluate the temperature rise in the soil surrounding multiple boreholes and their interaction. It is shown that the maximum temperature rise due to thermal interaction of multiple boreholes in a six-month period of heat transfer from the borehole into the soil occurs right after the beginning of the borehole (about 3% total length) and it decreases along the borehole length as the heat flux from the borehole wall into the soil decreases. Therefore, with the objective of limiting boreholes' operations and sizes in order to prevent their thermal interaction, the top length of the boreholes is the critical area. It can be concluded that using the constant heat flux method is only valid for the middle length of the boreholes and moving any further to the top or bottom of the borehole, the temperature rise evaluations become increasingly inaccurate.

5 Nomenclature

c_p	specific heat at constant pressure [J/kgK]
D	distance between the pipes in the borehole [m]
D_b	distance between the boreholes [m]
h	borehole distance from the coordinate centre [m]
h_z	integration variable [m]

H heating length
k soil thermal conductivity [W/mK]
k_b grout thermal conductivity [W/mK]
$\dot{m}$ mass flow rate [kg/s]
P dimensionless parameter
$\dot{q}$ generated heat per unit volume [W/m^3]
q' heat flux at borehole wall [W/m^2]
q'' heat flux at borehole wall [W/m^2]
r radial coordinate [m]
r_p pipe radius [m]
r_b borehole radius [m]
$\overline{R}_i$ dimensionless distance of Borehole i to a given point (x,y) in the solution domain
R_1 distance of Borehole 1 to a given point (x,y) in the solution domain [m]
R_2 distance of Borehole 1 to a given (x,y) point in the solution domain [m]
R_{11} thermal resistance between the inlet circulating fluid and the borehole wall [mK/W]
R_{12} thermal resistance between the inlet and outlet pipes [mK/W]
R_{22} thermal resistance between the outlet circulating fluid and the borehole wall [mK/W]
R_1^Δ dimensionless thermal resistance
R_{12}^Δ dimensionless thermal resistance
R_2^Δ dimensionless thermal resistance
R_p thermal resistance of conduction in the pipe [mK/W]
T temperature [K]
T_f' inlet circulating fluid temperature at z=100 m [K]
t time [s]
V volume [m^3]
Z dimensionless parameter
z axial coordinate [m]

Greek Letters

α thermal diffusivity [m^2/s]
β dimensionless parameter
Θ dimensionless temperature
φ circumferential coordinate [rad]
ρ density [kg/m^3]

Subscripts

$f1$ inlet circulating fluid
$f2$ outlet circulating fluid

6 Acknowledgements

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