

Simulink® Model of Single CO₂ Sensor Location Impact on CO₂ Levels in Recirculating Multiple-Zone Systems

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Abstract

Building ventilation can be modulated based on occupant requirements determined by monitoring CO₂ concentrations. These CO₂ concentrations are sometimes monitored in a common air duct to reduce costs. Warden (2004) suggested that a single CO₂ sensor located in the supply air duct (SACO₂-DCV) may provide better control of CO₂ in rooms with higher occupancy density than return air duct (RACO₂-DCV) placement systems serving multiple zones. Stanke (2010) pointed out that Warden failed to account for varying space population and airflow requirements. This research evaluated Stanke's observations using a hypothetical four-room-school model built with MathWorks®' Simulink®. The results showed that Warden's claim was valid for a dynamic environment but that SACO₂-DCV may over-ventilate spaces when occupancy varies. The simulation provided a graphical representation of the impact of CO₂ sensor location on room CO₂ levels when implementing CO₂-DCV in recirculating multiple-zone systems.

1 Introduction

Building designers strive to minimize energy use while meeting comfort and ventilation requirements. Modeling software can help by assessing whole building energy performance. Some issues must be analyzed beyond the limitations of whole building simulation programs to achieve more reliable results (e.g., thermal bridging in building envelopes). This paper addresses such an issue: the impact on zone CO₂ levels of placing a single CO₂ sensor in either the return or supply duct to control ventilation in a recirculating multiple-zone system (MZS).

DCV based on CO₂ monitoring has been recognized as an effective way of reducing energy use while maintaining acceptable indoor air quality (IAQ) where people are the main source of pollutants (Haghighat and Donnini 1992). CO₂-DCV can not ensure IAQ when other significant sources of pollutants are present. It is unclear whether a single duct-mounted CO₂ sensor averaging air from multiple spaces provides adequate ventilation (Murphy and Bradley 2008). At the time of writing, a research project funded by ASHRAE (RP-1547, "CO₂-Based Demand Controlled Ventilation For Multiple Zone HVAC Systems") is underway to verify whether CO₂-DCV for MZS avoids excessive CO₂ levels when used as an energy efficiency measure (ASHRAE 2011). Despite concerns regarding the CO₂ control effectiveness, this single sensor approach is commonly used in MZS to reduce costs as CO₂ sensors are expensive and require maintenance (Warden 2004).

When using a single CO₂ sensor in a recirculating MZS, Warden (2004) presented practical reasons why better CO₂ levels in the least occupied spaces would be achieved by monitoring CO₂ levels in the supply air duct (SACO₂-DCV) rather than in the return air duct (RACO₂-DCV). His analysis was based on an open-loop control. Warden, however, recognized his calculations, which were based on Standard 62-2001 (ASHRAE 2001), lacked validity for standards that have an area-based requirement for outdoor air (OA) such as Standard

62.1-2010 (ASHRAE 2010). As Stanke (2010) later pointed out, Warden's analysis also neglected the system response as population varies. Nassif et al. (2005), also researched the performance of SACO₂-DCV, focussing on fan energy use based on multiple space Equation 6-1 (ASHRAE 2001). The analysis used various analytical approaches including simulation work and on-site measurements. The authors concluded that SACO₂-DCV provided better CO₂-control than RACO₂-DCV.

To complement Warden and Stanke's work, this paper reports the results of a customized dynamic model to determine which of SACO₂-DCV or RACO₂-DCV provides better CO₂ levels across a multi-space environment when occupancy varies greatly. Unlike previous studies the area-based OA requirement was included in the model.

2 Methods

A model of an air handling unit (AHU) serving four classrooms was built to compare the impact of the CO₂-DCV control loop. Assumptions were made to limit the number of parameters involved to those necessary for a comparative assessment of the control system. The parameters necessary to create the initial RACO₂-DCV model were established based on a fictitious building. The SACO₂-DCV model was built modifying only the feedback loop and the controller gain of the RACO₂-DCV model. Simulations using both models were run to compare the variation of CO₂ levels in each room. Since this study focussed on the control system, keeping all elements of the building constant, Simulink® appeared to be the most appropriate tool. Simulink, is a graphical environment built on the well-known MATLAB®, which has been used for more than 15 years for various aspects of building and mechanical system simulation (Riederer 2005).

2.1 Assumptions

For the purpose of this research, the airflows to each zone, typically controlled by the zone thermal loads, were assumed to be constant, uniform and identical for all rooms. Using constant airflows significantly simplified the model to focus on the control of OA based on CO₂ levels.

The following is a list of additional assumptions made for simplicity sake:

- There were no external disturbances to the system.
- The air was well-mixed within each room.
- There was no stratification in the ducts.
- There were no short-cut between the supply and return air grills.
- There was no infiltration or ex-filtration in the rooms or ducts.
- The sensors were highly accurate and did not drift over time.
- The system had a zone ventilation efficiency of 1 (perfect mixing).
- Cool air was distributed through ceiling diffusers (zone air distribution effectiveness of 1.0, according to Table 6-2 of Standard 62.1-2010 (ASHRAE 2010)).
- The dampers worked perfectly and linearly in adjusting OA fraction.
- Occupants were the main source of pollutant.

2.2 Parameter Description

Parameters were selected to create a model based on proven equations to generate realistic results.

2.2.1 Room Configuration

The schematic diagram in Figure 1 shows 4 rooms served by a single AHU. The room floor area is typical for schools in the authors' province.

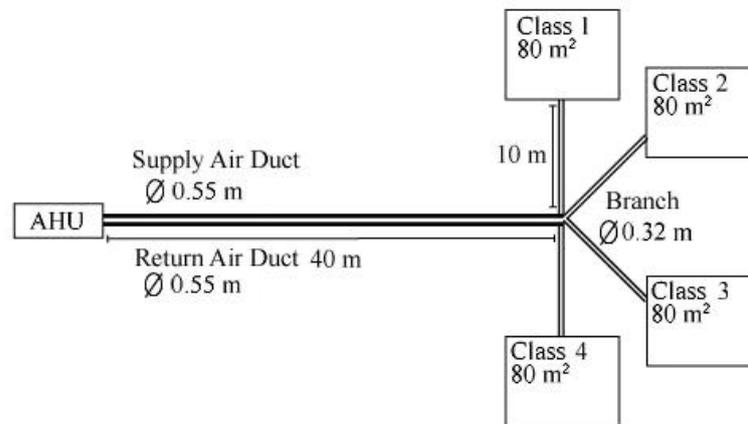


Figure 1 - Air distribution system schematic

The ceiling height was 3 m, giving a room volume of 240 m^3 . The spaces were assumed to be classrooms for students aged 9 and above. According to Table 6-1 of Standard 62.1-2010 (ASHRAE 2010), the default design occupancy density is 35 people/100 m^2 , yielding 28 students for 80 m^2 . The standard generates a per classroom OA requirement of 188 L/s ($5 \text{ L/s} \cdot 28 \text{ people} + 0.6 \text{ L/s} \cdot 80 \text{ m}^2$). The airflow to each room was based on the typical mixing system design supply air flow rate per unit floor area which is about 5 L/s m^2 , yielding 400 L/s per room. The system efficiency (Table 6-3 of Standard 62.1-2010) was 0.6, assuming 0.47 ($188 \text{ L/s} \div 400 \text{ L/s}$) is the minimum primary OA fraction. The design flow rate of OA required by Standard 62.1-2010 was therefore met.

2.2.2 Delays

Realistically, it takes time for air to circulate, which has been neglected in past analyses. This was represented by delays modelled in Simulink. Delays impact controller response and are system dependant, varying with duct layout, fans, controller algorithm and other disturbances.

As shown in Figure 1, the AHU where mixing occurs was approximately 40 m from the rooms. Assuming the 240 m^2 floor area was served by a duct width of 0.24 m^2 in cross section, the total duct volume from the room to the AHU was calculated to be 9.6 m^3 ($0.24 \text{ m}^2 \cdot 40 \text{ m} = 9.6 \text{ m}^3$). The total time for air to travel to and from the rooms was 8 s ($9.6 \text{ m}^3 / 1.2 \text{ m}^3/\text{s}$). Each room was served by a 10 m long duct with 0.08 m^2 cross section. The duct volume was 0.8 m^3 with design airflow of 400 L/s. The time in the smaller branch was therefore 2 s ($0.8 \text{ m}^3 / 0.4 \text{ m}^3/\text{s}$). The time for air to travel from one room to the AHU was 10 s. Since CO_2 sensors also take time to return a stable reading, 180 s delay was used for the feedback loop, based on a typically available sensor (GE Sensing 2006).

2.2.3 Occupant CO_2 Generation

The only input to the system was the number of occupants per room. Mumma (2004) provided evidence that equation (1) in difference form is accurate to determine the number of occupants in a room based on CO_2 concentrations.

$$P_z = \frac{\frac{V \cdot (C_r - C_{r-1})}{\Delta T} + V_{sa} \cdot (C_r - C_{sa})}{G \cdot 10^6} \quad (1)$$

Where:

G = CO₂ generation rate per person (L/s)

P_z = Number of occupants

V = Volume of the room (L)

ΔT = Time step (s)

V_{sa} = Supply air flow rate (L/s)

C_r = CO₂ level, current sampling time (parts per million (ppm))

C_{r-1} = CO₂ level, previous sampling time (ppm)

C_{sa} = CO₂ level, supply air duct (ppm)

In equation (1), the first term $(V \cdot (C_r - C_{r-1})) / \Delta T$, represents the transient state of the system, hence the time the system takes to reach its steady-state. The time step (ΔT) was chosen to 20 s.

Based on Lawrence (2008), the OA CO₂ concentration was constant at 400 ppm for all simulations. Therefore, initially, the rooms were assumed empty at 400 ppm. As time progressed CO₂ levels were calculated by rearranging the terms of equation (1) to form equation (2).

$$C_r = \frac{G \cdot 10^6 \cdot P_z + \frac{V}{\Delta T} \cdot C_{r-1} + V_{sa} \cdot C_{sa}}{\frac{V}{\Delta T} + V_{sa}} \quad (2)$$

Students in the classrooms were assumed sedentary (met = 1.0). The per person CO₂ generation rate was 0.0043 L/s as used by Warden (2004). This rate is also in accordance with Standard 62.1-2010 (ASHRAE 2010).

2.2.4 Controller

The same controller type (proportional) was used throughout the study in order to emulate the impact of positioning a given controller either in the supply or return air duct. As pointed out by Underwood (1999), this system was studied for a limited range of operating conditions for which the use of a constant gain controller was acceptable. The controller gain was tuned to achieve stability for both models and obtain the same steady state concentration as calculated with the concentration equation (Awbi 2003) shown in (3).

$$C_\infty = \frac{\mathcal{V} \cdot C_e + G}{\mathcal{V} + G} \quad (3)$$

Where:

C_∞ = Steady state concentration

\mathcal{V} = OA supply rate (L/s)

C_e = External concentration of pollutant

G = Volume of pollutant generated (L/s)

Using equation (3), given a CO₂ generation rate of 0.1204 L/s (0.0043 L/s · 28 occupants), an OA concentration of 400 ppm and an OA supply rate of 188 L/s, to meet the minimum ventilation requirements, resulted in a steady state concentration of 1039 ppm. This level is deemed acceptable according to Standard 62.1-2010 (ASHRAE 2010) which states “maintaining a steady-state CO₂ concentration in a space no greater than 700 ppm above OA levels will indicate that a substantial majority of visitors entering a space will be satisfied with respect to bioeffluents (body odor)”.

The controller sampling rate was set to 11 s for both controllers. The proportional band of the controller was set so the OA dampers start opening as the duct CO₂ levels are sensed above 400 ppm. The controller gains were adjusted so all rooms reached 1039 ppm at design occupancy to ensure adequate ventilation. The controller gains were 1/719 for SACO₂-DCV and 1/1357 for RACO₂-DCV.

2.2.5 Building Component

According to Standard 62.1 (ASHRAE 2010), a minimum ventilation rate per floor area is required and the outdoor airflow should be no less than this building component. CO₂-DCV does not measure airflow directly, which makes it difficult to establish whether the technique complies with ventilation standards (Stanke 2010). When the rooms were uniformly occupied, the model showed the building component was met. The building component in the current study was established as $0.6 \text{ L/s}\cdot\text{m}^2 \cdot 80 \text{ m}^2 = 48 \text{ L/s}$ per classroom. 48L/s is 12% of the primary airflow (400 L/s) hence a 12% opening of the OA damper.

2.2.6 Parameter Summary

The model parameters are summarized in Table 1.

Table 1 - Global simulation summary

Overall System	
OA CO ₂	400 ppm
Room Size	10 m x 8 m x 3 m or 240 000 L
Occupancy category	Educational facility for students age 9 and above
Supply air flow	400 L/s per classroom
Occupant CO₂ generation	
Time step	20 s
CO ₂ generation	0.0043 L/s per person
Delays	
Sampling time	11 s
Total RA delay	20 s (10 s each way)
Sensor air delay	180 s
Controller	
Type	Proportional
Controller scheme	SACO ₂ -DCV controller gain: 1/719 RACO ₂ -DCV controller gain: 1/1357
Min OA damper position	12%

2.3 Simulink Implementation

The previous section has established the origin of the parameters used in this research. The CO₂ generation in every room was based on equation (2), in which we inserted values of Table 1 to obtain equation (4).

$$C_r = \frac{1}{124} \cdot (43 \cdot P_z + 120 \cdot C_{r-1} + 4 \cdot C_{sa}) \quad (4)$$

Varying room airflows could be modeled by varying the V_{sa} value. If a variable air volume system (VAV) were to be simulated, the building ventilation component would have to be revisited. Nassif (2012) presented an approach of meeting Standard 62.1-2010 when the primary airflow varies.

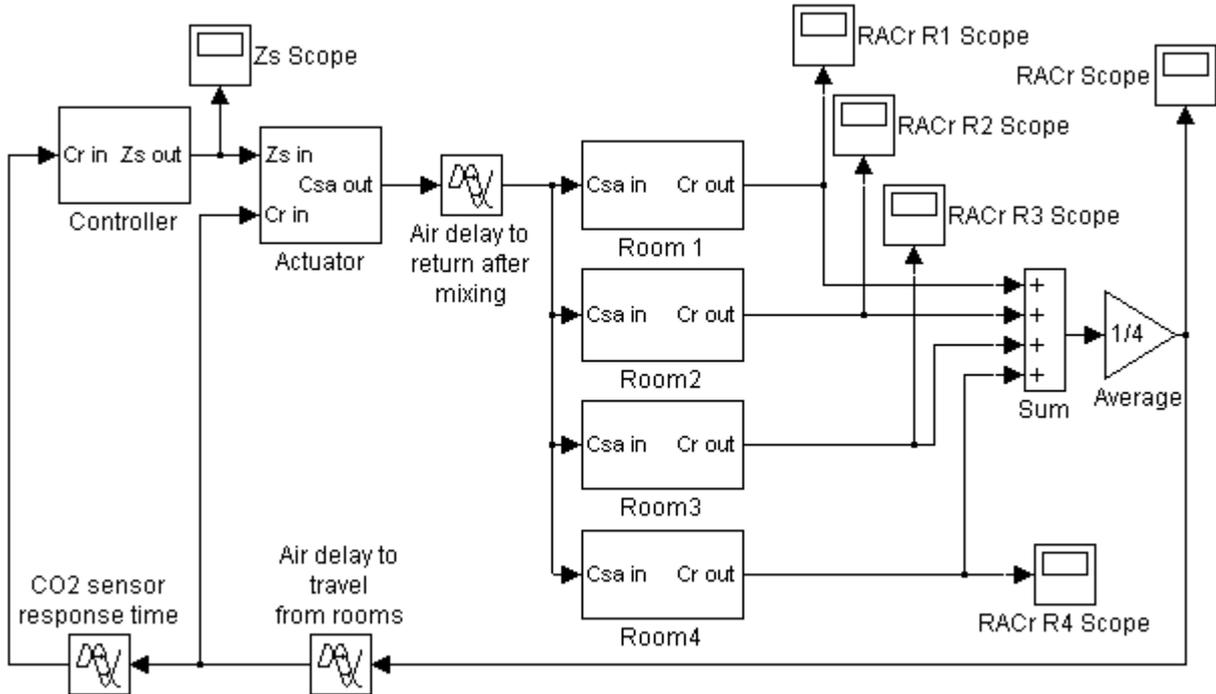


Figure 2 – Schematic of RACO₂-DCV Simulink model where C is the CO₂ concentration and Z_s is the opening ratio of the OA damper. Scope icons indicate virtual probe locations.

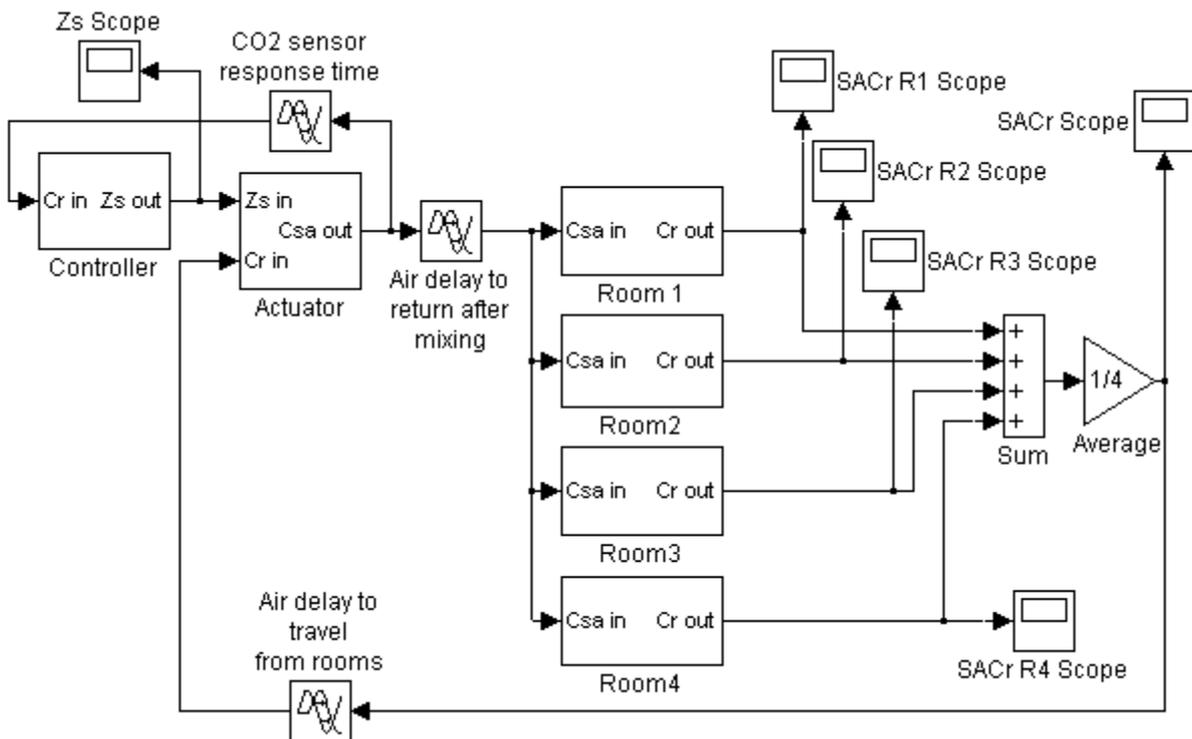


Figure 3 - Schematic of SACO₂-DCV Simulink model

The controlled variable in this project was the CO₂ concentration. The critical difference between these models was the sensor location, symbolized by the starting point of the feedback loop to the controller. Unlike the RACO₂-DCV strategy, the SACO₂-DCV feedback

loop returned the sensed CO₂ concentration from the supply air duct rather than from the return air duct. The total time for the air to recirculate to the room was the same in both cases. The delays were positioned to reflect sensors located near the AHU.

2.4 Simulation work

The models were validated to ensure Standard 62.1-2010 (ASHRAE 2010) minimum OA requirements were met when occupancy was uniform between the rooms. To do so, the OA damper opening (Z_s) was recorded at different occupancies. Z_s was multiplied by the air flow to each room (400 L/s) to determine the amount of OA supplied to each zone. The concentration equation (Awbi 2003) was rearranged to obtain equation (5) and validate the OA flow to each room.

$$V = \frac{G * (1 - C_{\infty})}{C_{\infty} - C_e} \tag{5}$$

The process was repeated for both models and compared to Standard 62.1-2010 (ASHRAE 2010) requirements.

The steady state room level CO₂ concentrations as population varied between the rooms were then assessed for both models. The goal was to test Warden’s claim that SACO₂-DCV provides better CO₂ control in rooms with higher occupancy density than RACO₂-DCV (Warden 2004). Equation (5) was used to assess if sufficient OA was delivered to each room based on the number of occupants.

Lastly, population was kept at design occupancy to observe the transient response of the room CO₂ levels. The goal was to determine if either system offered an advantage in the time taken to reach steady state.

3 Results

3.1 Uniform Occupancy

Room populations were kept uniform. The OA flow rates at different occupancies for the two Simulink models were compared to those based on Standard 62.1 (ASHRAE 2010) as shown in Figure 4.

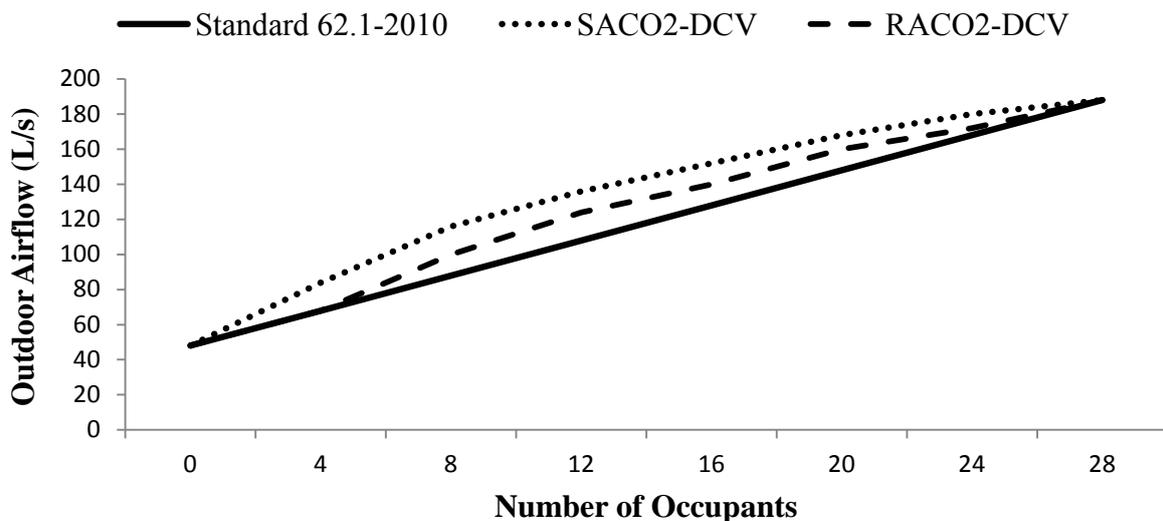


Figure 4 – System outdoor airflow versus occupancy for both models compared to Standard 62.1-2010 minimum requirements

The results in Figure 4 were obtained by reading the resulting Z_s value from each simulation and multiplying it by the airflow (400 L/s). Since the population was equally distributed among the rooms, the exact same airflows were found by inserting the simulation CO₂ concentration in equation (5).

Figure 4 shows both systems meet exactly the minimum required airflows when rooms are at design capacity or empty. Although both systems remain above the minimum requirements at intermediate capacities, RACO₂-DCV remained closer to Standard 62.1-2010 (ASHRAE 2010) requirements. When occupants are evenly distributed between zones RACO₂-DCV would therefore better meet Standard 62.1-2010 (ASHRAE 2010) ventilation requirements while reducing energy use. This is assuming the lower energy option is the one requiring less OA, as shown by Persily et al. (2003). SACO₂-DCV would maintain lower CO₂ concentrations by over-ventilating the spaces.

3.2 Partial Occupancy

Warden’s argument for the better performance of SACO₂-DCV was based on cases where population varies greatly between spaces. Multiple simulations were done to record CO₂ measurements as population varied as per Table 2.

Table 2 - Number of occupants per room for given simulations

Simulation Name	Room 1	Room 2	Room 3	Room 4
1-1-1-28	1	1	1	28
1-1-28-28	1	1	28	28
1-28-28-28	1	28	28	28

The number of people in each room was between full and low occupancy. Section 3.1 showed the minimum ventilation requirements were met at design capacity for all rooms and when rooms were empty. In this case, since occupants were unevenly distributed, Equation (5) was used to evaluate whether the outside airflow requirements were met.

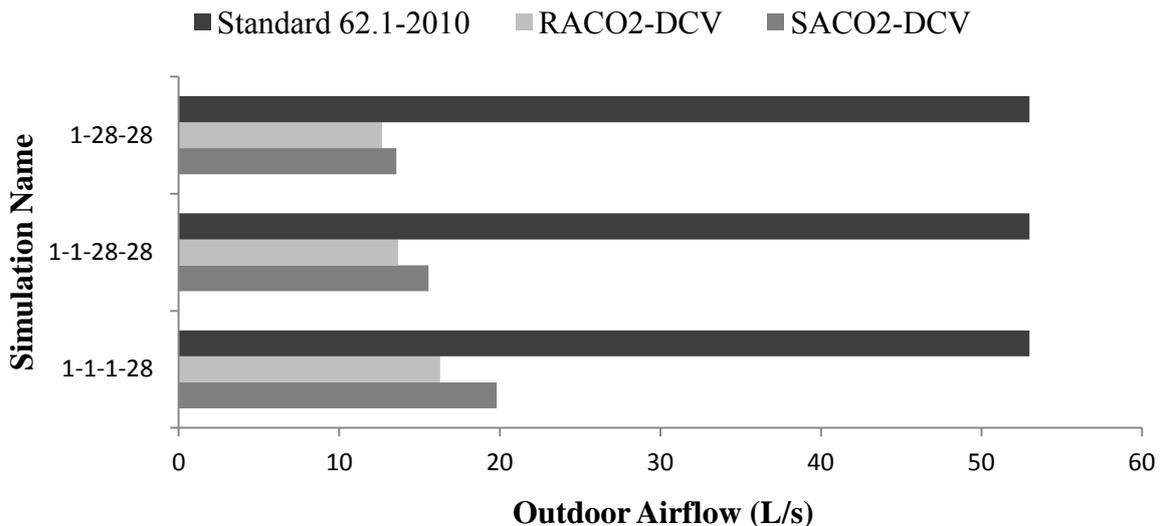


Figure 5 - Room 1 outdoor airflow as occupancy varies

Even though the simulations consistently showed room CO₂ levels below OA ppm+700 ppm as mentioned in 2.2.4, results in Figure 5 show the least occupied rooms re-

ceived much less than Standard 62.1-2010 requirements. In fact, the OA flow is much less than the minimum building component alone. As an example, for “1-28-28-28”, the damper opening, Z_s , would need to be opened close to 75% to meet the building component in the low occupancy room, causing the rooms at full occupancy to be largely over-ventilated. For rooms at lower occupancy, SACO₂-DCV provides marginally better CO₂ levels.

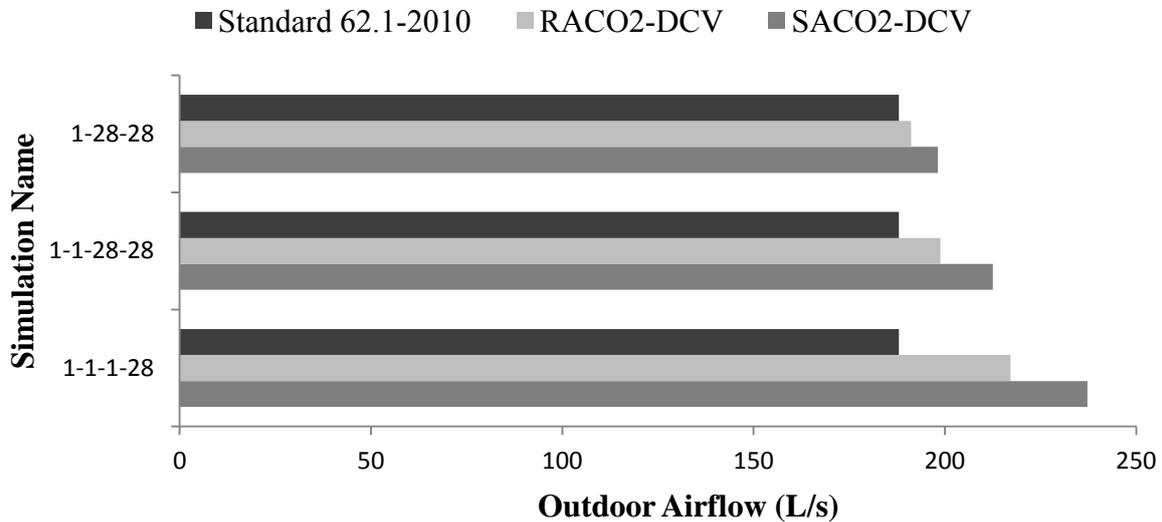


Figure 6 - Room 4 outdoor airflow as occupancy varies

Figure 6 correlates with and confirms Warden’s calculations (2004) showing that RACO₂-DCV leads to higher CO₂ concentration in a fully occupied room grouped with partially occupied rooms served by a common sensor than such a system with all rooms fully occupied. Differences between the two systems become more significant as population decreases. As per section 3.1, the RACO₂-DCV may, however, meet the minimum requirements with lower energy usage. SACO₂-DCV may result in over-ventilation.

It is important to note that a typical CO₂ sensor error is ±3% (GE Sensing 2006). At most, the two strategies for the same simulation show a difference of 45 ppm or 5%. Therefore, deviations noted are at the limit of sensor precision under real world conditions, especially assuming all potential errors linked to the model assumption were ignored in these results.

3.3 Transient Responses

At a constant population of 28 students in each room, the two systems perform as shown in Figure 7. Uniform occupancy led to the same CO₂ concentration, 1039 ppm, in every room as well as in the return air duct.

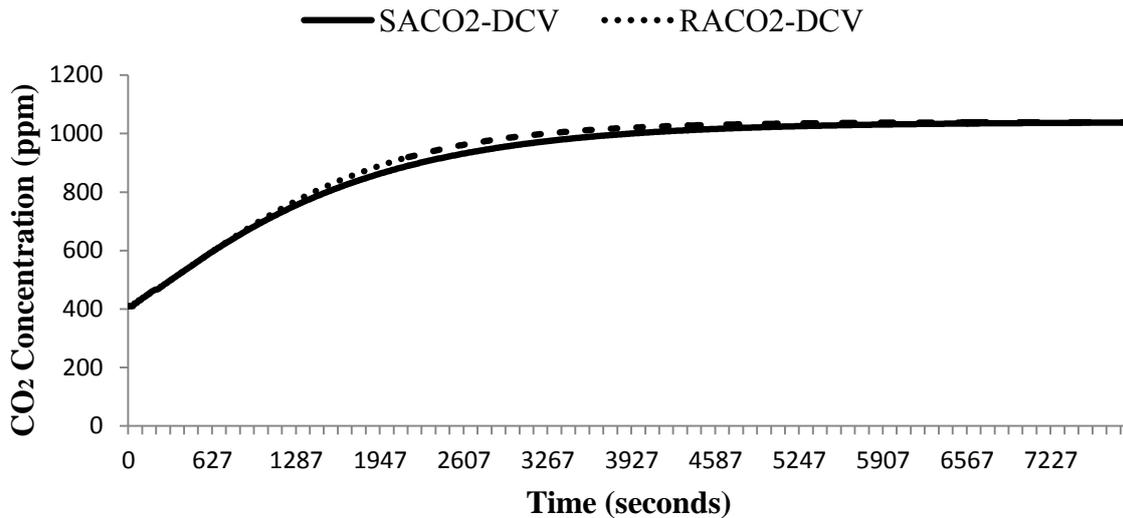


Figure 7 – Return air duct CO₂ concentration at design occupancy (28 students)

Figure 7 shows the transient state of both systems, a capability Warden’s (Warden 2004) static model lacked, as noted by Stanke (Stanke 2010). The figure shows both systems reached a steady state of 1039 ppm at design occupancy as per their controller setting. RACO₂-DCV however reached 90% of this value (935 ppm) in around 39 minutes while it took close to 44 minutes for SACO₂-DCV to reach the same value.

3.4 Discussion

The model was built to compare two design options independently of the building characteristics, since the building is assumed to be the same. Unlike other available tools such as CONTAMW used by Persily et al. (2003), Simulink offered the ability to create a customized model, tailored to the level of complexity required for the task. As an example, in the present case, the same theoretical CO₂ generation rate was used in both models. This rate did not account for the impact of airflow and heat transfer between zones or uneven contaminant mixing within a zone or duct. More advanced building simulation programs can consider these factors and are capable of creating much more comprehensive CO₂ generation rate algorithms. However, since both models use the same CO₂ generation rate algorithm, using the simpler algorithm would not alter the results of this comparative assessment.

The absolute numbers obtained through this study only approximated field conditions because of the assumptions mentioned above. Yet, the models can be used for comparison purposes, knowing the above assumptions would have the same impact on both models, apart from random disturbances.

4 Conclusion

This paper presented a Simulink model comparing the performance of SACO₂-DCV with that of RACO₂-DCV in recirculating multiple zone systems. Warden (2004), through a theoretical analysis, and Nassif (2005), through site measurements, both determined that SACO₂-DCV provides better CO₂ control than RACO₂-DCV when used in recirculating multiple zone systems. The Simulink model was used to verify this claim.

Unlike previous research, the dynamic model included a building component. This building component was set at design occupancy which revealed not to be sufficient when population varied. This study also showed that SACO₂-DCV may result in higher over-

ventilation of spaces than RACO₂-DCV in fully occupied rooms when the total population was below design occupancy.

Overall, the two systems were shown to have comparable performance given potential instrumentation error. Other factors such as cost and ease of access for installation and maintenance may determine the choice of sensor location. The concept of one system being superior to another is project-dependant and subject to designer goals and priorities.

5 Future Work

The advantage of the current model relies in its simplicity and adaptability. The model was built to answer a specific question without extensive knowledge about the building itself. This paper has shown Simulink can be a useful tool for CO₂-DCV analysis.

The following could be added to the current model:

- Assumptions outlined in section 2.1 could be added to the model. Disturbances could be modeled as additional delays.
- Co-simulation using this Matlab model with other existing simulation software such as CONTAMW may be an interesting avenue to study more complex CO₂-DCV questions involving the building envelope.
- Alternative controller algorithms could be implemented to analyze their performance.
- The model in this paper was relatively simple enough to be kept in the time domain. A time domain model is straightforward to modify and more readable even with little background in control system theory. The Simulink model could however be converted to the LaPlace domain to facilitate algebraic manipulation in more advanced analysis of the system.

6 Acknowledgements

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